XV. *The Application of Dr.* Saunderſon's *Theorem for ſolving unlimited Equations, to a* curious Queſtion in CHRONOLOGY: *By Mr.* James Horſefall, *F. R. S.*

Read March 24, 1768.

BY old tables it appears that Eaſter day happened on the 22d of March (which is the *ſooneſt* it ever can happen), in the years of Chriſt 165, 697, 1229, and laſtly in 1761.

Queſt. 1. What is the next year of our Lord, when it will happen ſo again before 1900? For,

Note. From thence to 2199, the paſchal full moon, or the golden number 14, which diſtinguiſhed the years above, will be fixed *on* the 22d of March; *conſequently* EASTER *day cannot happen before the 23d of March in that period.*

Anſwer. In the act for altering the ſtile, it appears by the table for finding Eaſter till 1899, that this can *never* happen in *that* period, but when the *golden number*, or lunar cycle, is 14, and the *Sunday* letter D.

Alſo, by making a *ſolar cycle* for that century, the firſt year of it will fall on 1812, the Sunday letters E D, wherefore all the years in that cycle, which have D for the Sunday letter, are 1, 7, 18, 24: and now the queſtion is reduced to this.

Queſt.

Queſt. What year of our Lord in the 19th century will have the ſolar cycle either 1, 7, 18, or 24, *when* the lunar cycle is 14? Or, which is the ſame thing;

Queſt. What number is there between 1800 and 1900, which divided by 28 leaves 1, 7, 18, or 24; and being divided by 19 leaves 14?

SOLUTION.

Here then in the general theorem $\frac{ar}{l} \times \overline{d - e} + d$, is $a = 28$, $b = 19$, $D = 1, 7, 18,$ or 24; $E = 14$, $l = 1$. To find r, the quotients are * 1, 2, 9; drop the firſt and laſt, becauſe their number is odd: then the ſeries required will be 0, 1 2; therefore $r = 2$.

* 19)28(1
9)19(2
1)9(9

N. B. If to any year of Chriſt be added 9, and the ſum divided by 28; the remainder, or 28, if 0 remains, will be the *cycle of the ſun* for that year; and if 1 be added to any year of Chriſt, and the ſum divided by 19; the remainder, or 19, if 0 remains, is the *cycle of the moon*. Hence, if any year of Chriſt be ſeverally divided by 28 and 19, and the remainders be d and e reſpectively; then $d + 9$, or $d + 9 - 28 = D$; and $e + 1$, or $e + 1 - 19 = E$. In the preſent caſe, taking $D = 1$, $d + 9 = 1$ cannot be; becauſe d would be *negative*: but d muſt *not only be affirmative*, but alſo GREATER than e; wherefore make $d + 9 - 28 = D = 1$; therefore

therefore $d = 1 + 28 - 9 = 25$; and $e + 1 = E = 14$, therefore $e = 13$.

D and E are the *cyclar* numbers, and *d* and *e* are the *anno domini* numbers ſuited to the theorem.

Here then $\overline{ar \times d - e} + d = \overline{28 \times 2 \times 7} + 20 = 412$; and $412 + 532 + 532 + 532 = 2008$. The firſt anſwer, therefore, A. D. 412, is too little, and when encreaſed by three *dionyſian* periods, or *multiples* of 28 and 19 is too big, going beyond the century required. So, *when this ſolar cycle is* 1, *it will not do.*

Let D = 7, the reſt as before. Then $d + 9 - 28 = 7$; therefore $d = 26$. Here then $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 13} + 26 = 754$; and $754 + 532 + 532 = 1818$. So A. D. 1818 WILL ANSWER THE QUESTION.

Let D = 18, the reſt as before. Then $\overline{d + 9 - 28} = 11$; therefore $d = 37$. Here $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 24} + 37 = 1381$; and $1381 + 532 = 1913$. This goes beyond the century required; ſo *will not do.*

Let D = 24, the reſt as before. Then $\overline{d + 9} = 24$; therefore $d = 15$. Here $\overline{ra \times d - e} + d = \overline{28 \times 2 \times 2} + 15 = 127$; $127 + \overline{532 \times 4} = 2255$; which goes *beyond* the century required.

So there is *but one year* in the 19th century, *viz.* 1818, that will have the conditions required. The cycle of the ſun will then be 7; the cycle of the moon 14; and the Sunday letter D; and *Eaſter Day the* 22*d of March*

N. B. For

N. B. For *every century* a *new* ſolar cycle muſt be made; becauſe, by the Act of Parliament* for correcting the calendar, every 100th year for three centuries is *common*, and *not biſſextile*; ſo that the ſame dominical letter ſtands againſt the ſame year, in the cycle only for 100 years in *three* ſucceſſive centuries.

N. B. By a continual addition of 28 to 1700 or 1756 †, we have the *firſt* year of each ſolar cycle; and when the *firſt* year of that cycle *next after the beginning* of any century is had, and its dominical letter found, by the rules and tables in the act, *the cycle for that century may be formed*, with the dominical letters anſwering to each year of it; whereby may be ſeen on *what* years of the cycle *the ſame Sunday letter recurs*. Thus;

Queſt. 2. If it was required to find in what years between 2200 and 2300 Eaſter Day would again happen on 22d of March; I find by the hints above, that the *firſt* year of the ſolar cycle falls on 2204; and, being *leap-year*, I find by the rules and tables in the act, that the *dominical-letters* are A G: from thence I conſtruct the ſolar cycle of 28 years, as in table I.

And from the table prefixed to the late Earl of Macclesfield's Letter to Martin Folkes, Eſq; P. R. S. read May 10, 1750, and publiſhed in Phil. Tranſ. Vol. XLVI. p. 47. ſhewing the place of the golden

* 24 George II.
† Vide Table I.

numbers in the calendar, and the paſchal full moon, and the Sunday letter, anſwering thereto for that century (which ſtand as in table II), I conſtruct table III, for finding Eaſter Day during *that* century; and obſerve it *never* happens on the 22d of March, but when the *golden number* C, and the *dominical letter* D.

And the dominical letter D happens only in the 4th, 9th, 15th, and 26th years of the ſolar cycle in that century, as appears by table I.

Now the queſtion is reduced to this, *viz.*

What number is there between 2200 and 2300; which, being divided by 28, leaves either 4, 9, 15, or 26; and being alſo divided by 19, leaves 6?

SOLUTION.

In the general theorem above, *viz.* $\frac{ar}{l} \times \overline{d-e} + d$ are given $a = 28$, $l = 1$, $r = 2$, as before; and to find the values of d and e,

We have $d + 9 - 28 = D = 4$; therefore $d = 23$ } in the theorem:
And becauſe $e + 1 = E = 6$; therefore $e = 5$ }

viz. $\overline{28 \times 2 \times 18} + 23 = 1031$; and $1031 + 532 + 532 = 2095$: ſo that this cyclar number will *not* do, the year falling either below or beyond the century required.

2. Let $D = 9$; the reſt as before. Then ſince $d + 9 - 28 = 9$; therefore $d = 28$, and $e = 5$ as before; and $\overline{28 \times 2 \times 23} + 28 = 1316$; and $1316 + 532 + 532 = 2380$. This cyclar number will *not* do, for the ſame reaſon as the laſt.

3. Let

3. Let D $= 15$; the reſt as before. Then ſince $d + 9 = \text{D} = 15$; therefore $d = 6$, and $e = 5$, as before; and $\overline{28 \times 2 \times 1} + 6 = 62$; and $62 + \overline{532 \times 4} = 2190$. This cyclar number will *not* do, for the ſame reaſon as before.

4. Let D $= 26$; the reſt as before. Then $d + 9 = 26$; therefore $d = 17$, and $e = 5$, as before; and $\overline{28 \times 2 \times 12} + 17 = 682$; and $689 + \overline{532 \times 3} = 2285$; and this is the *only* year that will anſwer the queſtion; becauſe it has 6 for its golden number, and D for its dominical letter. Whence we may conclude, that after A. D. 1761, there will not be ſo long a Trinity-vacation again till 1818; and after that year, the like will not happen till 2285.

Table I.
Solar Cycle, &c. for the 23d Century.

1756
112 = 4 × 28

1868
112

1980
112

2092
112

	Dominical Letters.	Years of the Sol. Cyc.
2204	A G	1
5	F	2
6	E	3
7	D	4
8	C B	5
9	A	6
10	G	7
11	F	8
12	E D	9
13	C	10
14	B	11
15	A	12
16	G F	13
17	E	14
18	D	15
19	C	16
20	B A	17
21	G	18
22	F	19
23	E	20
24	D C	21
25	B	22
26	A	23
27	G	24
28	F E	25
29	D	26
30	C	27
31	B	28
32	A G	1

Table III.
For finding Easter from 2200 to 2299, by the Golden Number and Dominical Letters.

G. N°	A	B	C	D	E	F	G
I	Apr. 16	17	18	19	20	21	22
II	Apr. 9	10	11	5	6	7	8
III	Mar. 26	27	28	29	30	31	25
IV	Apr. 16	17	18	19	13	14	15
V	Apr. 2	3	4	5	6	7	8
VI	Mar. 26	27	28	22	23	24	25
VII	Apr. 16	10	11	12	13	14	15
VIII	Apr. 2	3	4	5	Mar. 30	31	Apr. 1
IX	Apr. 23	24	18	19	20	21	22
X	Apr. 9	10	11	12	13	7	8
XI	Apr. 2	Mar. 27	28	29	30	31	Apr. 1
XII	Apr. 16	17	18	19	20	21	15
XIII	Apr. 9	10	4	5	6	7	8
XIV	Mar. 26	27	28	29	30	24	25
XV	Apr. 16	17	18	12	13	14	15
XVI	Apr. 2	3	4	5	6	7	1
XVII	Apr. 23	24	25	19	20	21	22
XVIII	Apr. 9	10	11	12	13	14	15
XIX	Apr. 2	3	4	Mar. 29	30	31	Apr. 1

Table II.
Lunar Cycle, &c. for the 23d Century.

Gold. Numbers.	D. of the Month for Paschal Full Moons.	Domin. Lett.
6	Mar. 21	C
	22	D
14	23	E
3	24	F
	25	G
11	26	A
	27	B
19	28	C
8	29	D
	30	E
16	31	F
5	Apr. 1	G
	2	A
13	3	B
2	4	C
	5	D
10	6	E
	7	F
18	8	G
7	9	A
	10	B
15	11	C
4	12	D
	13	E
12	14	F
1	15	G
	16	A
9	17	B
17	18	C
	19	D
	20	E
	21	F
	22	G
	23	A
	24	B
	25	C